Proceedings, Bicycle and Motorcycle Dynamics 2019
Symposium on the Dynamics and Control of Single Track Vehicles,
9 – 11 September 2019, University of Padova, Italy

Detailed Energy Flow Method for Analysis of Motorcycle Straight Running Stability

T. Katayama *, H. Tamoto #, T. Yoshino †, T. Kimura #

* Faculty of Mechanical Engineering
New Mongol Institute of Technology
43-1Manlaibaatar, 13372, Ulaanbaatar, Mongolia
e-mail: tkataya@kurume-it.ac.jp

Fundamental Technology Research Division
YAMAHA MOTOR CO. LTD
2500 Shingai, 438-8501, Iwata , Japan
e-mail: tsmotoh@yamaha-motor.co.jp

† Faculty of Engineering
Kurume Institute of Technology
2228 -66 Kamitsu, 830-0052, Kurume , Japan
e-mail: yoshino@kurume-it.ac.jp

Fundamental Technology Research Division
YAMAHA MOTOR CO. LTD
2500 Shingai, 438-8501, Iwata , Japan
e-mail: kimurate@yamaha-motor.co.jp

ABSTRACT

Detailed Energy Flow Method is proposed in order to discuss the mechanism of Wobble and Weave modes. First, the concept of the energy flow method is described. Before discussing the detailed method, it is shown that the yaw angle system is the main freedom for the weave mode. Next, detailed analysis methods are discussed, taking the stability change as the caster angle is increased as an example. The energy flow method proposed here demonstrates that the cause of the change in straight running stability can be understood in greater detail.

Keywords: motorcycle dynamics, straight running stability, energy flow method.

1 INTRODUCTION

In 1971, Sharp[1] published an analysis of straight running stability of motorcycles.
The research on the maneuverability stability of the motorcycle is making great progress with this work. It is clarified that the basic features of a two-wheeled vehicle can be expressed, considering four degrees of freedom with lateral velocity, yaw rate, roll angle and steering angle as dynamic variables.

It has been clarified that there are 3 types of unstable modes when traveling straight ahead.
Tease are well known cap size mode, wobble mode and weave mode. The motorcycle is unstable at the high speed range, the weave mode and the wobble mode become unstable, and may fall over a certain speed. It is important to understand the causes of these modes from a safety point of view. However, it had long been thought that the detailed elucidation of the causes are difficult using conventional Eigenvalue analysis.

A new method was proposed to understand the causes of the two vibrational modes[2],[3][2)-3)].
This method is named as Energy Flow Method. In this Energy Flow Method, the cause of the mode generation can be understood by calculating the energy flow by the force (torque) acting motorcycle.

In this paper, we show that the straight running stability analysis using the energy flow method can provide more detailed information than the findings obtained about 40 years ago.
More specifically, it is shown that the method proposed here can identify the terms of the equation of motion that governs the stability change.

This paper is organized as follows. First, review the basic concepts of the energy flow method. Before proposing a detailed energy flow method, we discuss the key degrees of freedom that govern the weave mode. Then we will introduce how to use the detailed energy flow method. In this discussion, we will use the change of caster angle, which is a well-known vehicle design specification, as an example. The last chapter is devoted to discussing the possibilities of this detailed energy flow method.

2 ENERGY FLOW METHOD USING EIGENVECTORS

First, the basic idea underlying energy flow method [2] is reviewed.

The time derivative of the kinetic energy of any system is given by the following equation.

$$d/dt(\,1/2\;\mathrm{m}\;\dot{x}^2\;) = \mathrm{m}\;\dot{x}\;\ddot{x} = \mathrm{m}\;\ddot{x}\;\dot{x}$$

where, m is the mass of the system, and $\dot{x}$, $\ddot{x}$ are the velocity and acceleration, respectively. On the other hand, as well known, the equation of motion of this system is expressed as follows.

$$\mathrm{m}\;\ddot{x} = F_1 + F_2 + F_3 + \cdot\cdot\cdot \qquad (2)$$

where, F_1, F_2, F_3 are forces acting the system.

Using equation (2), equation (1) can be rewritten as

$$d/dt(\,1/2\;\mathrm{m}\;\dot{x}^2\;) = \mathrm{m}\;\ddot{x}\;\dot{x} = (F_1 + F_2 + F_3 + \cdot\cdot\cdot)\;\dot{x}$$
$$= F_1\;\dot{x} + F_2\;\dot{x} + F_3\;\dot{x} + \cdot\cdot\cdot \qquad (3)$$

The meaning of equation (3) is that the time change of kinetic energy can be known by the product of the individual forces acting on the system and the velocity.

That is, when $F_1\;\dot{x}$ is positive, it means that kinetic energy is increased by the force F_1. This can be interpreted as the force F_1. promoting the motion.

Applying this idea to the wobble mode, it can be interpreted as follows.

It is possible to know the generation mechanism of the wobble mode by multiplying the force acting on the steering system by the steering angular velocity, since the wobble mode is a vibration of the steering system. For example, when $F_1\;\dot{x}$ is positive, the force F_1 excites the wobble mode, and when $F_2\;\dot{x}$ is negative, F_2 suppresses the wobble mode.

There are two methods of calculating energy flow. One is a method using a numerical solution of the equation of motion. The other one is to use eigenvectors. The simulation method is intuitive and easy to understand, but it takes time to calculate.

Calculations using eigenvectors have been shown to give exactly the same results as the simulation method [2]. Here, an energy flow method using eigenvectors is described.

The wobble mode is taken as an example, in order to help understanding easily. It is well known that the wobble mode is a vibration of the steering system.

Therefore, we use the equation of motion (4) for the steering system.

$$(I_{fz} + M_f\,e^2)\;\ddot{\delta} = -M_f\,e\ddot{y}_1 - (M_f\,ek + I_{fz}\cos\varepsilon)\ddot{\psi} - (M_f\,e\,j + I_{fz}\sin\varepsilon)\ddot{\phi} - (M_f\,e + i_{fy}/R_f\sin\varepsilon)$$
$$\dot{x}_1\dot{\psi} + i_{fy}/R_f\cos\varepsilon\dot{x}_1\dot{\phi} - (t\,Z_f - M_f\,eg)\;\phi - (t\,Z_f - M_f\,eg)\sin\varepsilon\delta - tY_f - \cos\varepsilon T_{zf} - \sin\varepsilon T_{xf}, \qquad (4)$$

Here, the symbols I_{fz} and M_f represent the moment of inertia and mass of the front frame, and e, j and k are the distance from the steering axis, the height, and the front and back distance of the center of mass of the front frame, respectively. Symbol t is Mechanical trail, ε is caster angle. The mechanical variables of the four degrees of freedom are $\dot{y}_1$,ψ,ϕ,δ,which represent lateral velocity, yaw angle, roll angle and steering angle, respectively. In terms of tire force, Y_f, T_{zf} and T_{xf} represent the lateral force, aligning torque and overturning torque of the front tire, respectively. The dot above the symbol of a variable represents the time derivative of that variable. Equation (4) is the Newton's second law, so the right side terms are the torques acting on the steering system. However, in the energy flow method, these are simply called "forces". See references [1] and [2] for the complete set of equation of motion.

In the energy flow method using eigenvectors, the eigenvectors are substituted into the part corresponding to the variables of the above equation (4). The eigenvectors are given by magnitude and phase with reference to a certain eigenvector. Therefore, when eigenvectors are substituted into Equation (4), the forces are represented by a two-dimensional vector, as shown in Figure 1. Here, the steering angular velocity vector is used as a reference. The arrows shown by solid lines in Figure 1 represent the forces acting on the steering system. According to the energy flow method [2], the horizontal X component of these forces is proportional to the real part of the eigenvalues, and the proportional coefficient is the coefficient $I_{fz} + M_f\,e^2$ on the left side of Equation (4). Also, the vertical Y component is proportional to the imaginary part of the eigenvalue (proportional coefficient is $I_{fz} + M_f\,e^2$). The sum of these forces is the arrow shown by the broken line in the figure. It is possible to know the activation mechanism of the wobble mode, by examining at the X component of each force.

In the example shown in Figure 1, it can be seen that the wobble mode is mainly excited by yaw rate force and roll rate force, on the other hand, suppressed by lateral acceleration force and front tire force.

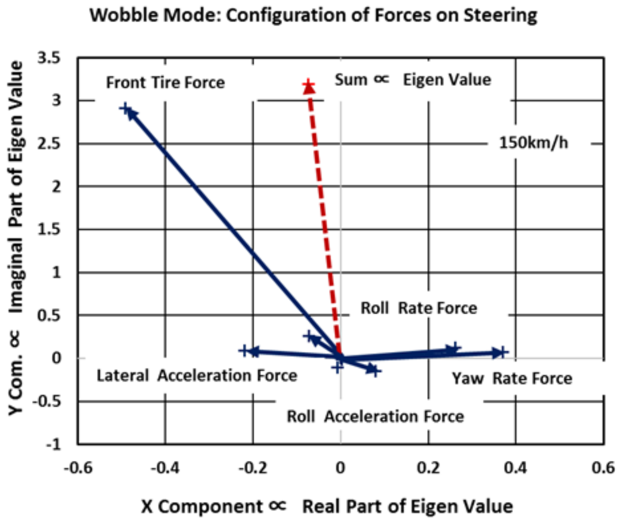

Figure 1. Configuration of forces acting on steering system

3 DOMINANT DEGREE OF FREEDOM FOR WEAVE MODE

As is well known, the wobble mode is considered to have the steering system as the main freedom because of the relative magnitudes of the eigenvectors.

On the other hand, weave mode is believed to be a complex vibration mode in which four degrees of freedom were coupled. However, according to reference [3], it is concluded that the degree of freedom of the yaw angle system affects the weave mode most strongly. It is recognized that the results derived from this point of view fit well with the experience of engineers involved in motorcycle design.

The dynamical supporting evidence is shown here. In general, the equation of motion that describes harmonic vibration is given by $\mathrm{m}\ddot{x} + kx = 0$. As well known, the frequency is known to be proportional to $(\mathrm{k}/\mathrm{m})^{1/2}$. This idea is applied to the four-degree-of-freedom model, in order to specify the degree of freedom that greatly affects weave mode. That is, the coefficients of the acceleration term in the left side of equation of motion are independently changed, and the change of the eigenvalue is examined. Then, we compare the change of the Eigen value imaginary part with the prediction result in the one degree of freedom theory. It is hypothesized that the most matched degree of freedom is the dominant degree of freedom for the weave mode.

This hypothetical correctness is made for the wobble mode before the discussion to the weave mode. For example, the lateral motion freedom is examined as follows. The lateral acceleration term of the equation of motion with respect to the lateral freedom is increased by 10%. Then, the imaginary part of the eigenvalue at that condition is calculated in order to compare with the expected eigenvalue, which is $(1 / 1.1)^{1/2}$of the original value. The considerations for the other three degrees of freedom are done in exactly the same way.

Figure 2 shows the comparison of the coefficient change of the lateral motion system. The comparison of the yawing system is shown by Figure 3. Figure 4 is a comparison of rolling freedom. Figure 5 shows for steering freedom. In these figures, horizontal axis shows vehicle running speed. The original imaginary parts of Eigen value are shown by black circles with solid line. And the white circles with solid line show the imaginary part of Eigen values after changing the coefficient. The predicted values are indicated by the triangular marks connected by a broken line.

It can be understood from Figure 5 that the change of the imaginary part of the eigenvalue when the steering acceleration term of the steering system is changed agrees well with the predicted value.

However, it can be seen from Figures 2-4 that the results of changing the other three freedoms do not show good agreement with the predicted value compared with Figure 5. These results are consistent with the conventional interpretation. Thus, we can consider that the hypothesis introduced above is justified.

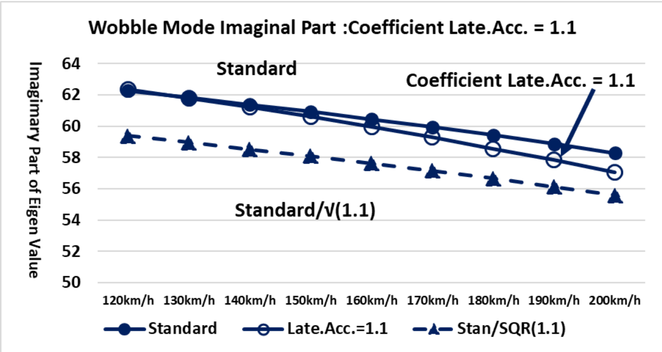

Figure 2. Imaginary part of Wobble Mode Eigen Value when changed Lateral acceleration coefficient

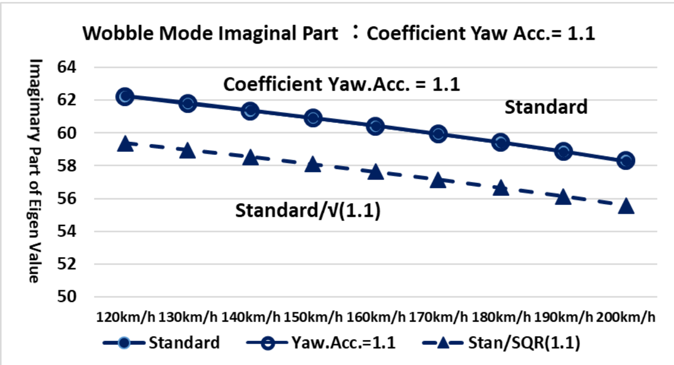

Figure 3. Imaginary part of Wobble Mode Eigen Value when changed Yawing acceleration coefficient

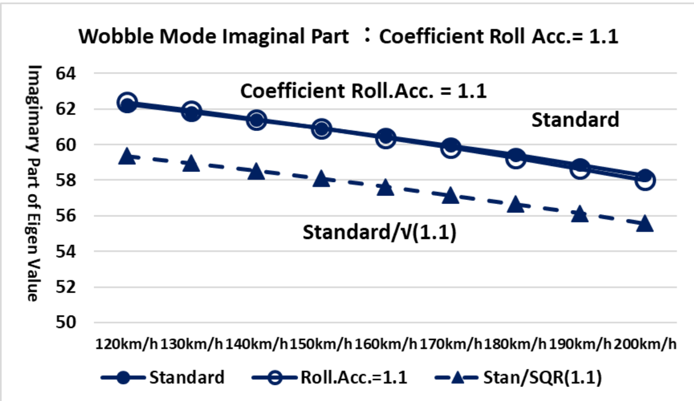

Figure 4. Imaginary part of Wobble Mode Eigen Value when changed Rolling acceleration coefficient

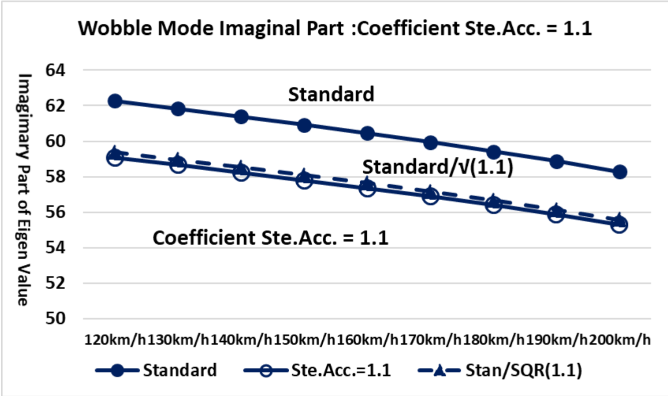

Figure 5. Imaginary part of Wobble Mode Eigen Value when Steering acceleration coefficient

The examination for weave mode are shown in Figures 6-9. The format of the Figure is the same as in Figures 2-5. Figure 6 shows a comparison for lateral freedom, where the change in the eigenvalues is calculated when the coefficient of the acceleration term in the lateral motion system is increased by 10%, and compare the Eigen value with the predicted value. Figures 7-9 are comparisons of yawing, rolling and steering freedoms respectively.

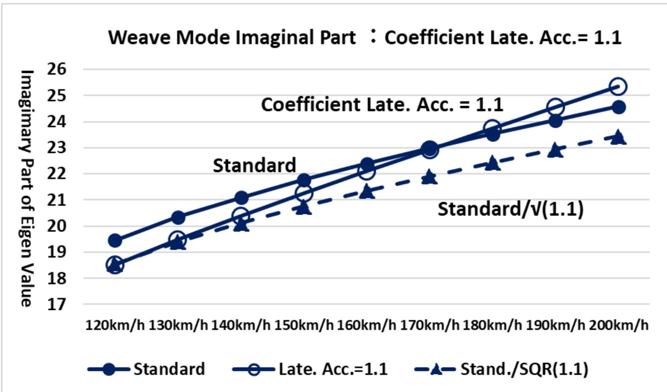

Figure 6. Imaginary part of Weave Mode Eigen Value when changed Lateral acceleration coefficient

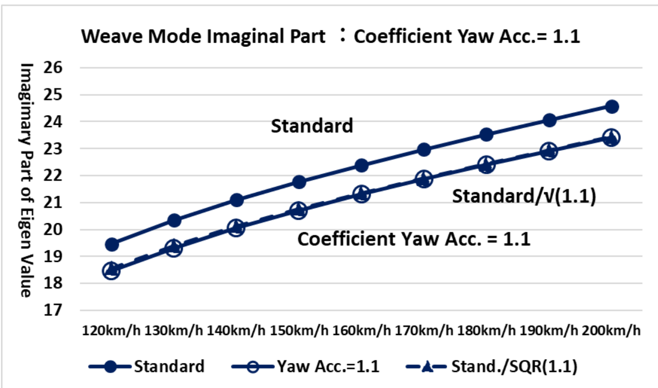

Figure 7. Imaginary part of Weave Mode Eigen Value when Yawing acceleration coefficient

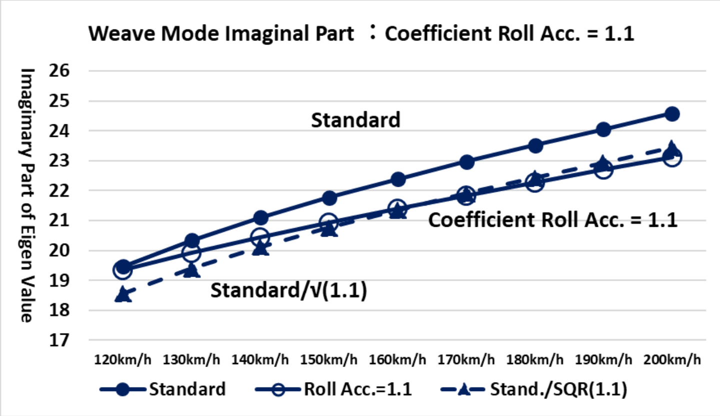

Figure 8. Imaginary part of Weave Mode Eigen Value when Rolling acceleration coefficient

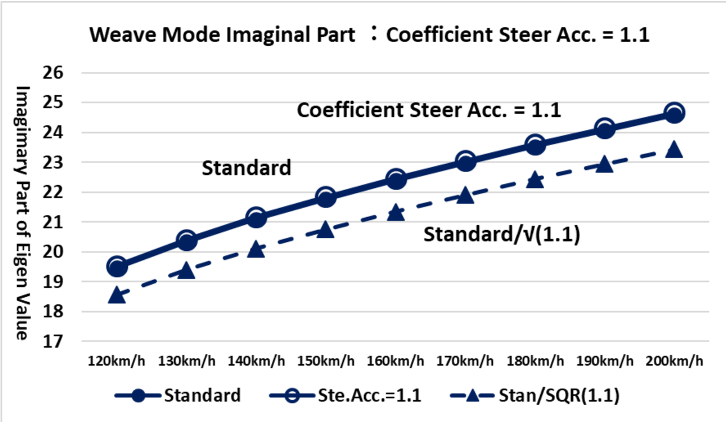

Figure 9. Imaginary part of Weave Mode Eigen Value when Steering acceleration coefficient

From these figures, it can be said that coefficient change of yaw angle acceleration of yawing system shows very good agreement with predicted value. However, the other 3 freedoms do not show a good agreement, compared with yawing freedom.

These discussions show that the weave mode is to be main vibration of the yaw angle system, and give the theoretical support for conventionally claimed [3] and for the experience of the designers and producers.

4 CALCULATION PROCEDURE OF DETAILED ENERGY FLOW METHOD

As is it is well known that the stability of straight running changes due to changes in designing parameters. In this chapter, we will describe the procedure that can identify the cause of Eigen value change, tracing back to the original equation.

In the method proposed here, we first understand the cause in a large scale. Then, we examine the detailed energy sequentially and finally clarify the cause factor by means of only one term in equation of motion. The method consists of four major steps following above idea, as shown in Figure 10 and Figure 11. The energy involved in characteristics can be divided into two. That is, it is divided into energy due to the force of the vehicle body interaction, internal forces, and the one due to the force from the outside of the vehicle, external forces. Here, tire force is taken as the external force.

In the first step, energy is divided into two parts, one is due to the interaction between body movements, and the other is due to the tires. Then, it is examined which is the cause of the energy change (Figure 10).

The second step is to identify the most contributing elements or freedoms(Figure 10). For example, in the case of in-vehicle Body energy, one degree of freedom that makes the largest contribution is extracted from the four degrees of freedom. In the case of tires, it is examined whether the contribution of the front tire is larger or the contribution of the rear tire is larger.

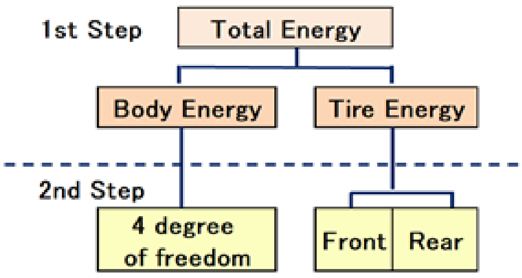

Figure 10. Analysis procedure: first and second steps

In the third step, as shown in Figure 11, more detailed contribution is identified. For example, taking the degree of freedom of yaw as an example, we examine whether the yaw acceleration or the yaw rate contribution is greater. Furthermore, it is examined whether the cause is due to the change in magnitude of the force vector or the phase of the vector. When the case is due to phase, more detailed examination of the contribution is not possible. Therefore, a detailed study on the magnitude of the force vector is advanced (Figure 11).

At the beginning of the fourth step, it is confirmed whether the cause of the magnitude of the force vector is due to the coefficient of the equation or the change in the magnitude of the eigenvector (upper part of 4th Step stage in Figure 11). Also, more detailed discussion of the magnitudes of eigenvectors is not possible.

Thus, it is finally clear which part of the relevant coefficients of the equation are responsible for the change of the eigenvalues (lower part of 4th Step stage in Figure 11). This method is described next by taking the case where the caster angle ε is increased by 20% as an example.

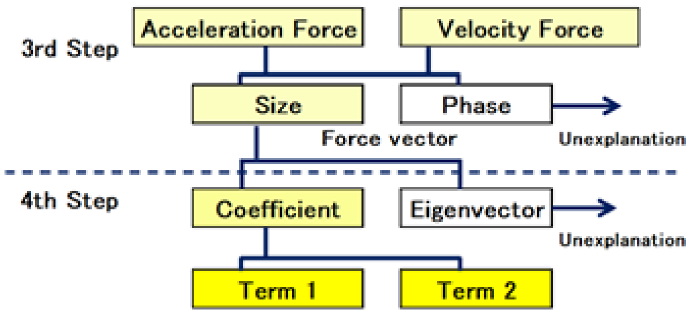

Figure 11. Analysis procedure: 3rd and 4th steps

4.1. Application to Wobble Mode

As is well known, the wobble mode becomes unstable and the weave mode becomes stable when the caster angle is increased. Here, the method proposed above is used to identify the causes. The wobble mode is considered first.

In the first step, energy change of steering freedom is shown in Figure 12. It can be seen that when the caster angle is increased, the total energy increases and wobble mode becomes unstable (positive energy change). The main cause is the increase of the energy of the body system. Conversely, the energy of the tire system decreases, and it is understood that the tire system contributes to stabilization.

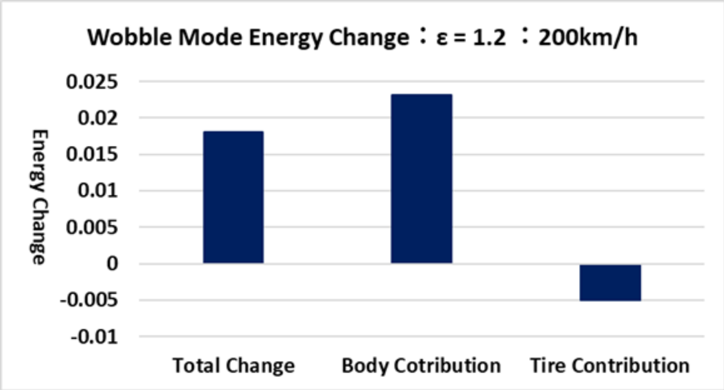

Figure 12. Energy change of steering freedom: Wobble Mode

In the second step, Figure 13 shows that the contribution of the yaw freedom is the largest among four degrees of freedom.

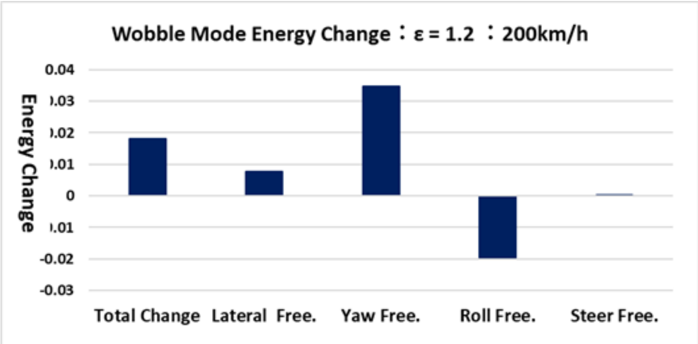

Figure 13. Detailed energy change of steering freedom: Wobble Mode

Next is the third step. In the energy contribution of the yaw angle freedom shown by Figure 14, the contribution of the yaw rate is positive and larger than the contribution of the yaw angular acceleration. According to Figure 15, examining the contribution of yaw rate, it is understood that the contribution due to the magnitude change is larger than the contribution of the phase change. So analysis is continued.

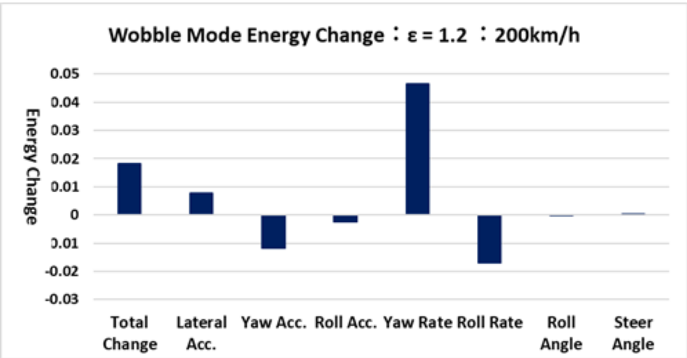

Figure 14. Energy change contributions of steering freedom: Body contribution

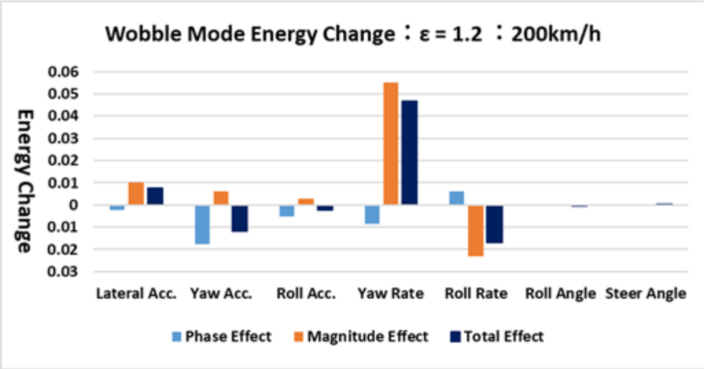

Figure 15. Phase and Magnitude effects of acting force on steering freedom: Wobble Mode

The last is the 4th step. The next thing to consider is whether the cause is due to changes in the equation coefficient or eigenvector. It is necessary to investigate the change of coefficient of equation and the change of eigenvector of yaw rate. The changes in these values are shown in Figure 16. In this figure, the coefficient change is shown on the left side, and the right side is the change in the magnitude of the eigenvector of the yaw rate. It can be understood from Figure 16 that increasing the caster angle increases the yaw rate force, which is due to the fact that the coefficient of the equation is increased. That is the coefficient of the equation is larger (1.1 times) than the magnitude of the eigenvector (1.02 times).

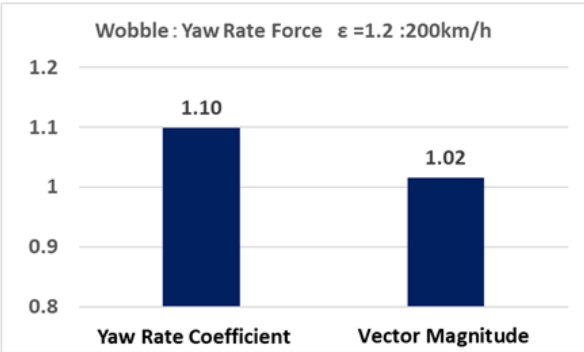

Figure 16. Coefficient and Eigen Vector change of Yaw Rate force acting to steering system

It can be seen that the coefficient of the corresponding equation is $(M_f e + i_{fy}/R_f \sin\varepsilon)\ \dot{x}_1$ of the fourth term on the right side of equation (4). And it is clear that the corresponding term is $i_{fy}/R_f \sin\varepsilon$. It is a so-called gyro moment. That means, it can be understood that the cause is that the term $\sin\varepsilon$ becomes large.

The results of the above study are summarized as follows.
(1) Wobble Mode destabilizes when the caster angle ε is increased.
(2) The cause is the force by the motion of the motorcycle body.
(3) In more detail, this is because the yaw rate force acting on the steering system increases.
(4) The reason for the increase in the yaw rate force is that $i_{fy}/R_f \sin\varepsilon$ is contained in the coefficients of the equation.
(5) That is, it is caused by the increase of the gyro torque of the front wheel.

4.2. Application to Weave Mode

Weave mode can be considered as well. The weave mode is stabilized by the energy contribution of the tire system, as can be seen from Figure 17.

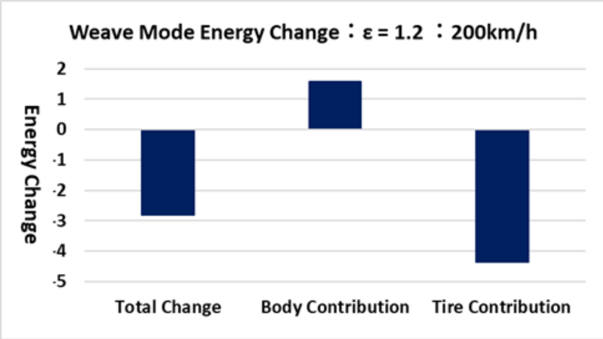

Figure 17. Energy change of yawing freedom: Weave Mode

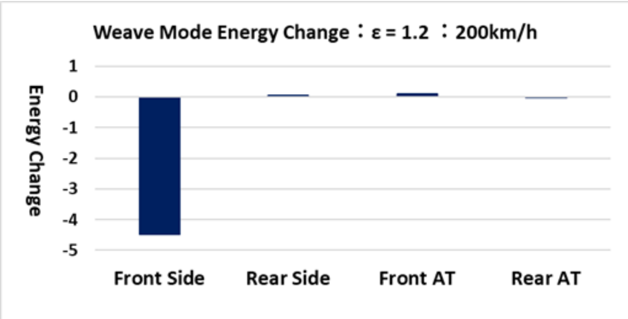

Figure 18. Detailed energy change of Tire contribution: Weave Mode

Examination of the detailed force component in Figure 18 shows that the contribution of the side force of the front tire is the largest. Next, we discuss the details of the front tire side force. The equation for the front tire side force is given by the following equation.

$$\mathbf{Y_f} = -C_{Ys1}/\dot{x}_1\,\dot{\boldsymbol{y}}_1 - C_{Ys1}l_1/\dot{x}_1\,\dot{\boldsymbol{\psi}} + C_{Ys1}t/\dot{x}_1\,\dot{\boldsymbol{\delta}} + C_{Yc1}\varnothing + (C_{Ys1}\cos\varepsilon + C_{Yc1}\sin\varepsilon)\,\boldsymbol{\delta} - \sigma_f/\dot{x}_1\,\dot{\boldsymbol{Y}}_{\mathbf{f}} \quad (5)$$

where C_{Ys1} and C_{Yc1} are cornering power and camber stiffness of front tire respectively. And σ_f is relaxation length of front tire side force.

The result of substituting Eigenvector of weave mode into the variable part of this equation is shown in Figure 19.

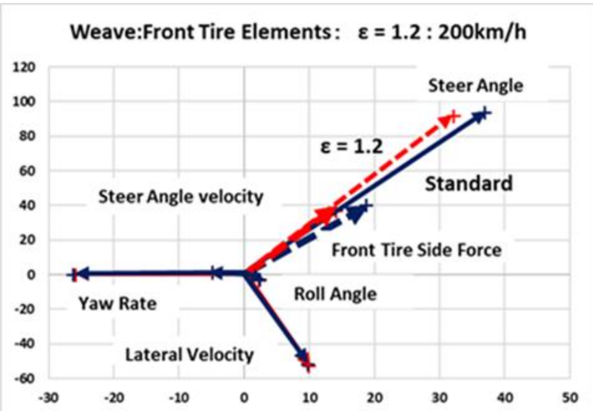

Figure 19. Configuration of front tire side force elements: Weave Mode

It is possible to understand how each element contributes from this figure.

In the figure, increasing the caster case is represented by red arrows. The X component of each vector directly contributes to the stability of the weave mode. For example, the yaw rate element contributes the most to the stability, and the steering angle component leads to destabilization.

It is also known that the steering angle element (shown by red broken arrow) changes because the phase advances and the size is reduced, when the caster angle is increased.

Figure 20 shows this more quantitatively. From figure 20, we see that the weave mode is stabilized by the change in magnitude at the same time as the phase advances, as shown by the component part of the steering angle. The effect of advancing the phase is stronger about 3.5 times of the magnitude effect (3.77 / 1.1 = 3.42).

This situation is shown in more detail in Figure 21. The phase advance of the steering angle element is due to the phase advancing of the steering angle eigenvector.

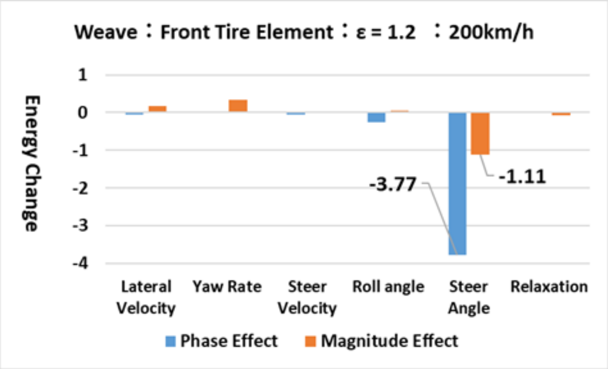

Figure 20. Phase and Magnitude effects of front tire side force elements: Weave Mode

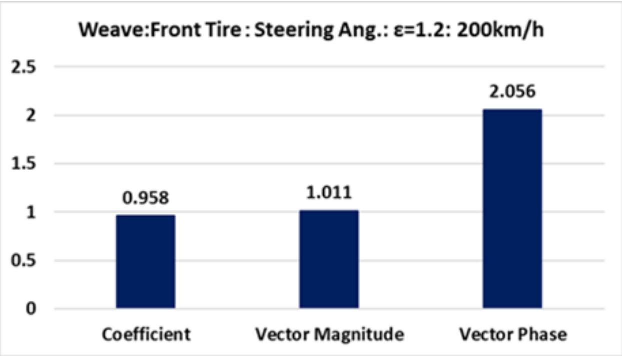

Figure 21. Coefficient and Eigen Vector change concerning steer angle element of front tire side force: Weave Mode

It can be also seen that the magnitude change is due to the change in the coefficient of the equation and is not due to the change in the magnitude of the eigenvector. The coefficient of the steering angle element corresponds to the fifth term on the right side of equation(5), that is $C_{Ys1}\cos + C_{Yc1}\sin\varepsilon$. This coefficient contains two factors concerning caster angle. It is easy seen that the factor whose value decreases as the caster angle increases is $C_{Ys1}\cos\varepsilon$. This term is cornering power.

The results of the study of weave modes are as follows.

(1) Weave Mode stabilizes when the caster angle ε is increased.

(2) The cause is tire force, that is the front tire side force.

(3) It is due to the fact that the phase of the front tire side force advances and at the same time the magnitude decreases.

(4) The change in the phase and magnitude of the front tire force is mainly due to the change in the steering angle element.

(5) The phase change and the size change of the steering angle element also occur.

(6) The change in the phase of the steering angle element is caused by the phase change of the steering angle Eigenvector.

(7) The magnitude of the steering angle element can be attributed to the coefficient $C_{Ys1}\cos\varepsilon$.

4 CONCLUSIONS

Heretofore, it has been thought that the mechanism that governs the straight running stability of a two-wheeled vehicle is not clear in the Eigenvalue analysis. However, as discussed here, it has become clear that the activation mechanism etc. of the two vibration modes of a two-wheeled vehicle can be elucidated by using the energy flow method with Eigenvectors.

The energy flow method can be applied to various situations. For example, it is applicable to solve the high speed instability of weave mode and wobble mode. As is well known that the characteristics of high-speed weave mode have not been deeply understood until 50 years after Sharp's Eigenvalue analysis.

The present study is a discussion using the most basic model of a two-wheeled vehicle, so these findings are unlikely to immediately contribute to the development of everyday two-wheeled vehicles. However, this method is expected to enable basic interpretation of the characteristics of a two-wheeled vehicle dynamics.

REFERENCES

[1] R. S. Sharp, “The stability and control of motorcycles”, *Proceedings of the IMechE, Part C, Journal of Mechanical Engineering Science* **13** (1971), pp. 316-329.

[2] T. Katayama and T. Nishimi, “Energy Flow Method for the Study of Motorcycle Wobble Mode”, *Vehicle System Dynamics* **19**(1990), pp.151-175.

[3] Y. Marumo and T. Katayama, “Analysis of Motorcycle Weave Mode by using Energy Flow Method”, *Journal of Mechanical Systems for Transportation and Logistics* **2**(2009), pp.157-169